# Problem-set 2

## Winning strategies in two-player games

**Problem 1.** *In Lecture 3 and Theorem 1.10 of the notes, the following thereom is stated.*

**1.10.** *For any **two-player, zero-sum** game with **perfect information** and **no chance**, that ends after a finite number of moves, one of the following is true.*

1. *Player 1 has a winning strategy;*
2. *Player 2 has a winning strategy;*
3. *Player 1 and Player 2 both have strategies which ensure a draw.*

*Are all the assumptions needed? For each assumption, give a counter-example where the assumption is dropped and the theorem is false, or else an argument that the assumption is not needed.*

1. *Two-player;*
2. *zero-sum;*
3. *perfect information;*
4. *no chance.*

## Equilibria in normal form games

**Problem 2.** *Below is a pay-off matrix for a zero-sum, two-player game with perfect information and no chance. The two players are Alice and Bob. Alice has three actions: A–C; Bob has four actions: a–d. The table shows the pay-off to Alice.*

| | | *Bob* | | | |
|---|---|---|---|---|---|
| | | *a* | *b* | *c* | *d* |
| | *A* | -2 | 0 | -3 | 1 |
| *Alice* | *B* | -1 | 1 | 1 | -1 |
| | *C* | -2 | -1 | 2 | 3 |

(a) *Find a Nash equilibrium for the game using* dominance. *If it takes more than one step to get to the equilibrium, be sure to write down every step in reducing the matrix until the equilibrium is found. (Remember, a dominant strategy for Player 2 has lower values than other strategies, for all opponent actions.)*

*(b) Now find the equilibrium using the minimax approach, again giving the steps in your reasoning.*

**Problem 3.** *This question involves the following game. The two players move simultaneously.*

| | | Brian | |
|---|---|---|---|
| | | *a* | *b* |
| *Amy* | *A* | $(1, 1)$ | $(-1, -1)$ |
| | *B* | $(-1, -1)$ | $(3, 2)$ |

*(a) What type of game is this? Name all types of games it is*

*(b) Find all of the pure strategy equilbria. Justify that they are equilibria.*

*(c) Is there a fully mixed strategy equilibrium? If so, find it. Otherwise, give a convincing argument that one does not exist.*

## Game trees

**Problem 4.** *Figure 1 shows a game tree. In what order are the nodes values determined? Nodes can be evaluated left to right or right to left, but say which you are using.*

**Problem 5.** *Figure 2 shows a win-loss tree. Apply win-loss pruning (either left to right or right to left). What nodes are* not *evaluated.*

**Problem 6.** *Figure 3 shows a game tree. Apply alpha-beta pruning (either left to right or right to left). What nodes are* not *evaluated. What are the final alpha-beta values of nodes C1, C2, C3 and the root?*

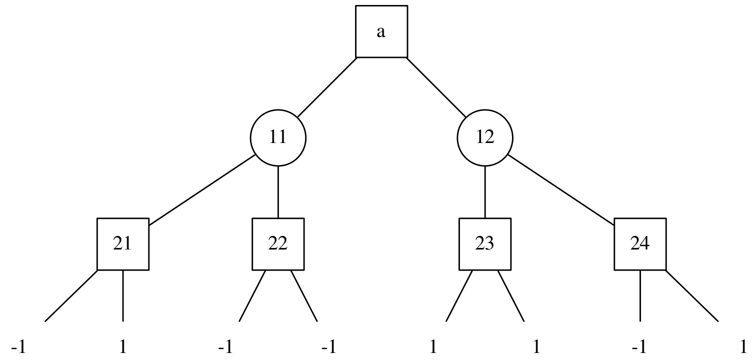


Figure 1: A game tree used for Problem 4. Squares are `MAX` nodes and circles are `MIN` nodes.

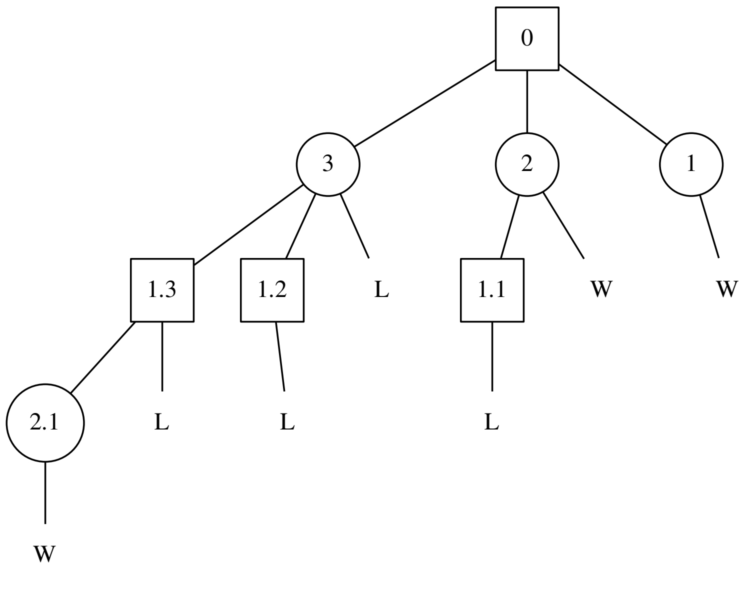


Figure 2: A win-loss tree associated with Problem 5. You might recognize this tree.

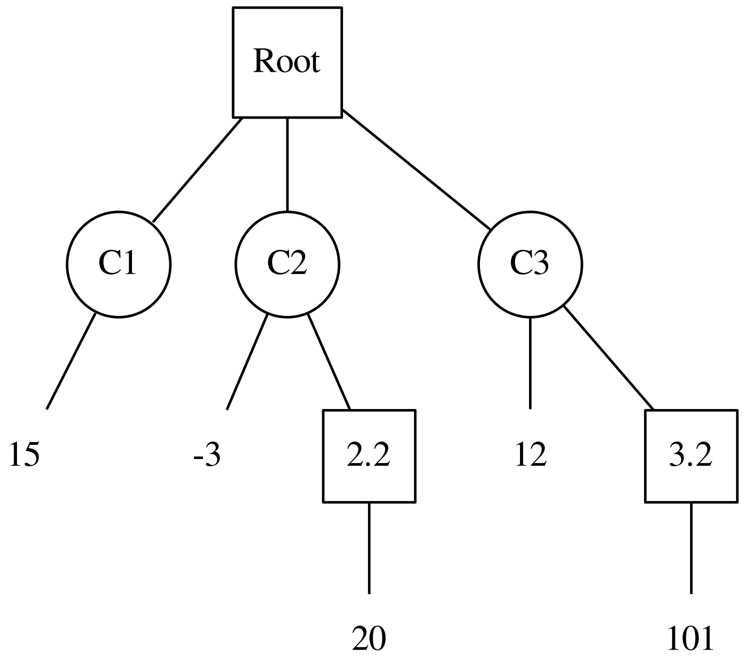


Figure 3: A game tree associated with Problem 6. To practice alpha-beta pruning. You might recognize this tree.