

Forty-five minutes

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

14/11/16

Time: 12.00

This is a CLOSED book examination

The use of electronic calculators is not permitted.

1. a) Consider the following function:

$$\begin{aligned} \mathbb{C} &\longrightarrow \mathbb{R}^+ \\ z &\longmapsto z \cdot \bar{z} \end{aligned}$$

Is this function injective? Is it surjective? Justify your answers. (5 marks)

b) Consider the binary operation on the set

$$\{a, b, c\}$$

given by the following table.

\otimes	a	b	c
a	a	a	a
b	a	b	c
c	c	c	c

Is this operation associative? Is it commutative? Justify your answers. (5 marks)

2. a) i) Construct a truth table for the formula: (3 marks)

$$(P \wedge \neg Q) \leftrightarrow \neg(\neg P \vee Q).$$

ii) Determine if the formula is a tautology. Explain your answer. (1 mark)

b) Answer **one** of the following: (2 marks)

i. Briefly explain **one** key difference between the Boolean semantics and the power set semantics of propositional formulas.

ii. Give **two** reasons why transformation to conjunctive normal form is useful.

c) Use our CNF algorithm to transform this formula into conjunctive normal form. Simplify your answer as much as possible. (4 marks)

$$(P \wedge \neg R) \rightarrow \neg(Q \rightarrow \neg(P \rightarrow Q)).$$