

# Exam Performance Feedback Form

## COMP11120, Semester 1

2018/2019

First some general remarks. 220 students sat the exam. The median mark was 41 out of 60 (or 68%), and the average 37.7 (63%). This was significantly better than last year's results, and we felt overall that students knew the material better. See below regarding the mistakes that were typically made.

Overall we were happy with the results. The greatest worry is the number of students who were unable to gain marks for the probability question.

Twenty-six students had a failing mark, that is, a mark of less than 24 out of 60, or a mark of less than 40%. Of these, seven students had a mark below 18 (which is below 30%), with a lowest mark of 14. At the top end, ninety-two students achieved first class marks with a top mark of 58, which is 97%.

Statistical analysis of individual questions:

**Question 1.** The average for Question 1 was 62%, which is a huge improvement on last year. Only 118 students offered any answer for this question. Twenty-nine students had a failing and ninety-one students a first class mark. Two student achieved full marks.

In general it was very good to see that a lot of students were able to make coherent arguments using mainly the English language rather than being fixated on using symbols. In some cases their statements lacked bits of detail for complete rigour, but these often gained almost all the available marks.

**Question 2.** This had an average mark of 58%, slightly below last year. This question had a comparative large number of students who could answer almost none of the parts correctly, and a large number of students who got almost everything correct. Again only 118 students offered an answer to this question. There were sixty-eight student on a failing mark and 104 students on first class mark, with eight students getting full marks.

Twenty-nine students got a mark of at most four out of twenty, and it makes us wonder whether some students did not take the material on probabilities, which was taught towards the end, all that seriously. Certainly attendance in the examples classes tailed of significantly for this material.

**Question 3.** Overall average: 70.2%; 23 students got 18-20 out of 20 (2 got all 20).

### Question 1.

- a) The vast majority of students were able to use the definition of divisibility to work out that they had to look at the remainder for modulo  $i$ , or straightforwardly write the given numbers as multiples of  $i$ , either of which is fine. Students who insisted on using fractions lost marks, because division is not defined for integers. To arrive at the multiples new variables have to be introduced, and if it wasn't clear from what was written that this is an existential statement, the writer would lose a mark. (So it's okay to say 'we can find  $m \in \mathbb{Z}$  such that  $im = j$ ', but it's not okay to write 'so  $j = im, m \in \mathbb{Z}$ '.) The majority of students got two out of the three available marks due to this mistake.
- b) This was very well answered by the vast majority of students. The given operation is both, commutative and associative, and almost everybody gained the mark awarded for a giving a reason for commutativity. When it came to establishing associativity a few students used wrong statements,

such as ‘the operation is the same as conjunction for booleans’, or ‘three symbols evaluate to 1 if they’re all equal’. If such an incorrect way or reasoning was used, all three marks for the justification were lost. The most popular correct answer was to give all eight cases, but other possibilities do exist (for example reasoning that three symbols evaluate to 1 if and only if the number of 1s in the input is odd).

- c) The vast majority of students were able to answer that the given function is not surjective, and to also give a correct counterexample, with a reason that it is a counterexample, gaining all three available marks for that part. Suitable counterexamples are the strings 0 or 1, strings of odd lengths, or strings ending in 0, none of which appear in the range of the given function.

The function given is injective, and it was very good to see a lot of sensible arguments written in English, with students not worrying much about using symbols. A lot of the given arguments had minor details missing (for example when it came to removing the trailing 1s from some output of the function), but many students gained at least some of the three marks available for the argument.

Some students claimed that the function had an inverse and concluded that it had to be a bijection. What these students actually did was to construct a partial inverse in line with Proposition 2.2—they did not consider the option that the input to their ‘inverse’ might be for example the string 0, or 1.

- d) Note that the question only asked about which function eventually dominates which, and it was not necessary to give a reason, or calculate from when one function dominates another. The function (ii) eventually dominates (i) which eventually dominates (iv) which eventually dominates (iii). If function  $f$  eventually dominates function  $g$  which eventually dominates function  $h$ , then function  $f$  eventually dominates  $h$ , so it is not necessary to say more, but we didn’t make that explicit. Many students wrote something for each pair of functions, which is fine.

The most common mistake was to claim that (iv) eventually dominates (i), which is incorrect. Other than this, a few students were confused about ‘eventually’, and only wrote about domination (which was not asked), and some students clearly didn’t understand the notation because they assumed one of the given functions was  $n \mapsto n$ , despite the fact that this notation is used consistently in the notes and was the subject of exercises in Week 1. No marks were available for comparing the given functions with the identity. Some students missed some of the valid instances of the asked for relation, and the number of marks awarded depended on how many they failed to give.

**Question 2.** A lot of students gained very high marks on this question, but a disturbing number of students only got very low marks.

- a) I asked specifically for a probability space. This consists of three components, and many students lost marks because they only give one or two of these. Students might want to take note that writing  $\mathcal{E} \subseteq \mathcal{P}S$  does *not* specify  $\mathcal{E}$ , it merely gives a necessary condition. Similarly, some students wrote  $P: \mathcal{E} \rightarrow [0, 1]$ , which does not tell me which probabilities are assigned to which outcomes or events and so does *not* define a probability distribution. Students who are preparing for future instances of this exam are well advised to look at how probability spaces are described in examples in the notes, or in the solutions to the exercises.

Some students were confused about the possible outcomes, these can either be described as  $\{0, 1, 2, 3\}$  or  $\{00, 01, 10, 11\}$ , but I don't think I saw any other correct version.

- b) There were plenty of correct answers for this question. The outcomes 0 and 3 (or 00 and 11) now have probability  $2/6$ , while the outcomes 1 and 2 (or 01 and 10) now have probability  $1/6$ .

Some students gave 'distributions' where the probabilities didn't add up to one (using  $2/8$  and  $1/8$ ), or they gave distributions where it wasn't the case that the second bit being different from the first was half the probability of them being equal (using probabilities  $3/8$  and  $1/8$ ). Lastly a small number of students gave the probabilities the wrong way round.

- c) I asked for a new distribution *for the space from b)*, and a lot of students failed to give probabilities for *all* the available outcomes. The correct answer assigns probability 0 to 00 and 01, probability  $1/3$  to 10 and probability  $2/3$  to 11.

Not relating the given answer to the original outcomes lost a mark. There were some mistakes in the calculations, and some students seem to have confused about what it means to express a number in binary—confusing 01 and 10. Students who had the wrong answer to b) but then consistently calculated in a correct manner could get full marks for this and all the following questions.

- d) Many of the same points from the previous part apply. The correct answer assigns probability 0 to 00 and 10, probability  $1/3$  to 01 and probability  $2/3$  to 11.

- e) The simple way of answering this is to calculate the expected value of the random variable of running the program once, which is  $3/2$  for both distributions, and arguing that we need twice that value since  $E(Y + Z) = EY + EZ$  for random variables  $Y$  and  $Z$ . Many students instead calculated the distribution of the given random variable, which was quite error prone.

A few students forgot to double the result from running the programme once, and lost a mark that way. Some students didn't seem to know what an expected value is, giving an answer other than a number, which gained 0 marks.

- f) Bayesian updating is explained in great detail in the notes. In this year's question I made it explicit that there are two options we are trying to distinguish between (see the Tip in the notes), and the majority of students realized that this means we want to update a distribution which has two possible outcomes. This makes it a bit easier to carry out the calculations than for questions where there are more available options.

The updates (when using the correct distribution from b)) are  $(3/7, 4/7)$ ,  $(9/17, 8/17)$  and  $(27/59, 32/59)$ .

In order to get all ten marks for the final part of the question I expected students to give as their conclusion that it is more likely that the given program implements the distribution from b), but that the difference between the two probabilities is not large enough to be very confident in this.

Failing to add a caveat lost a mark, and merely restating exactly the final update gained neither of the two marks for this bit. Some students seemed to get lost in the course of calculating updates and the marks awarded were in line with how far they got. Students who had wrong numbers in b), and who used those to correctly carry out Bayesian updating, could in principle get full marks if their calculation was not simpler than the one intended.

Some students failed to identify their updates and these were all but lost in the jumble of calculations, and in some cases this lost a mark or two. If there were then mistakes in the calculations, but no account of what a student was trying to do, I could only judge how far the student had got in making the correct updates.

**Question 3.** Overall the performance on the logic question was up from last year, which was very pleasing to see. The hardest question was d) on first-order logic where the quality of the answers was mixed.

a) Average mark: 82%. 47 students got full marks.

In each case a mark was given for the correct answer (either **true** or **false**) and one mark for a correct explanation of the answer.

The question posed few problems. Most got all the answers right. Marks were lost for minor mistakes and vague, imprecise or absent explanations.

Everybody knew what tautologies and contradictions are but there was some misunderstanding over satisfiable formulas. A satisfiable formula does *not* mean there must be a valuation where the formula is 0. A formula is satisfiable if *at least one valuation assigns it is 1*, which means that all tautologies are satisfiable. The answer to ii) is therefore **true**.

Despite knowing the answer, many could not explain iv) or did not specify how truth tables can be used to validate a judgement. In propositional logic the validity of the judgement can be checked by construction a truth table for the corresponding implication and exploiting Theorem 3.11. This implication needed to be given.

b) Average mark: 69%; 53 students got full marks.

Overall this question was answered well.

- (i) The transformation to CNF required just replacement of  $\leftrightarrow$ ,  $\rightarrow$ , application of De Morgan's law. Few mistakes were made in this part. Replacing  $P \leftrightarrow Q$  by  $(\neg P \wedge \neg Q) \vee (Q \wedge P)$  was not the best choice since it led to unnecessarily long calculations. It would have been better to use the law which says that  $P \leftrightarrow Q$  is equivalent to  $(P \rightarrow Q) \wedge (Q \rightarrow P)$ . This is the standard definition of  $\leftrightarrow$  and the one given in the notes. The CNF did not need to be simplified.
- (ii) This required transformation of the CNF obtained in i) to DNF and simplified as far as possible. Most common were mistakes in the application of distributivity (Step 4 or distributivity laws). The correct final answer was  $\neg P \wedge \neg Q$ . Some students gave  $\neg(P \vee Q)$  as a final answer, though equivalent is not in DNF, so a mark will have been lost.

Marks were lost for:

- not saying which rules are applied in all steps or using wrong rule names (e.g. just writing 'simplify' is not sufficient; contrapositives or absorption were wrongly given as a justification a few times). It is ok to write out the rules used instead of giving the name (which many students did).
- small mistakes such as
  - \*  $Q$  becomes  $R$  (due to poor handwriting)
  - \* simplifying  $\perp \vee A$  to  $\perp$  (this is wrong)
  - \* omitting brackets in e.g.  $P \vee \neg Q \wedge \neg P \vee Q$  and consequently getting confused. This formula has no meaning, since according to our bracketing convention  $\wedge$  and  $\vee$  have equal priority.

c) Average mark: 79%; 156 students got full marks

Overall this question was answered well. I was pleasing to see how many students got full marks. Marks were lost for:

- not stating what rules were applied to which premises (not often)
- not noticing that for the conjunction introduction rule the assumptions of both premises must match. If we have
  1.  $P \vdash P$  and
  2.  $P, Q \vdash Q$conjunction introduction cannot be applied. Applying Weakening to the first judgement we get
  3.  $P, Q \vdash P$ .Now conjunction introduction can be applied to 2 and 3 to get 4.  $P, Q \vdash P \wedge Q$ .
- Applying  $\rightarrow$  introduction without discharging an assumption.  $\rightarrow$  intro is not applicable to  $P \vdash P \wedge Q$  If we want to end up with the implication  $Q \rightarrow (P \wedge Q)$  on the RHS of the turnstile,  $Q$  must be an assumption on the LHS of the turnstile. From  $P, Q \vdash P \wedge Q$  we can derive  $P \vdash Q \rightarrow (P \wedge Q)$  by application of  $\rightarrow$  Introduction. Note how  $Q$  on the LHS of the turnstile becomes the LHS of the implication.

A very small minority of students could not construct a ND proof at all.

d) Average mark: 40%; 9 students got full marks

The quality of the answers were mixed and on the whole a bit disappointing. Of the four available marks, 9 students got all 4, 45 got 3, 54 got 2 and 46 got no marks.

Common difficulties and mistakes:

- Since there is no mention of students or staff in the first or second sentence, the predicate symbols  $S$  and  $St$  should not appear in the answers.
- not being able to work out what the quantifiers should be and leaving one or both variables unquantified.
- in ii) the existential quantifier must precede the universal quantifier
- wrongly using the given symbols. For example since  $B$ ,  $I$  and  $S$  are predicate symbols it is wrong to write either  $B(y, I(x, Ref))$  or  $B(S(y), I(x, Ref))$ . Only variables, constants and function symbols may occur inside the arguments of predicate symbols! Some also used the  $Ref$  and  $JRL$  as variables, but these are constants and cannot be quantified.
- getting the order of the arguments of  $I$  or  $B$  the wrong way around.
- in iii) implications in the wrong direction.

General tips for doing these types of questions:

- Try to simplify the translation task, by translating parts of the sentence first, or first considering the sentence in a simplified form (e.g. "Borrowing items is restricted to staff and students" in the case of iii).
- Double check your solution by translating your proposed solution into English and check if this expresses the same information as the given sentence.
- Double check the quantifiers, double check the order of the quantifiers, double check the order of arguments of the predicates, double check the number of arguments.