

Exam Performance Feedback Form

COMP11120, Semester 2

2016/2017

First some general remarks.

225 students sat the exam. The average was 57% (or 34 out of 60 marks). Sixty-six students achieved a first class mark, with a maximum of 57 out of 60. Thirty-three students had a failing mark, with 18 of these a mark below 18, which is less than 30% of the available total, and in turn 6 of those had a mark below 10.

If we look between the extremes than forty-four students had a mark from 40% to 49%, forty-seven from 50% to 59%, and thirty-five one from 60% to 69%.

Compared with last year's exam that's a lower average, although we felt that the quality of answer was improved. It was more the case that students made mistakes due to misreading the question, making mistakes in calculations or overlooking a key aspect of what was asked rather than lacking in understanding.

Statistical analysis of individual questions:

Question 1. The average for Question 1 was 61%. Twenty-nine students had a failing mark, and thirteen of those did not manage more than two out of twenty marks. One hundred and six students a first class mark and sixteen of those achieved full marks.

Question 2. This had an average mark of 53%, with twenty-seven students on a failing mark, of which eight received no more than two marks. Fifty-seven student had a first class mark. The highest mark achieved was nineteen out of twenty, which only one student managed.

Question 3. This had an overall average mark of 58.4%, with 15 students on a failing mark. 125 students had a first class or upper second class mark (with 73 students on a first class mark). 11 students got 18-20 out of 20 marks of which 1 student achieved full marks.

Question 1. Overall the answers to this question were good, with part b) being particularly well answered over all.

- a) This was well answered by most. The majority of those who had trouble with this question made a mistake when calculating the first few values, using the formula, inserting $(n + 1)$ rather than n in the given formula, so their first few values were 5, 8, 13, 20, ... whereas the actual values are 5, 6, 9, 14, 21. There was a surprisingly large number of students who made this mistake, which meant that they couldn't recover to answer the remainder of this part in a sensible way. Very few students appear to have gone back to check these numbers when they couldn't complete the question.

The function required here is

$$n \longmapsto n^2 + 5.$$

Some students wrote an expression involving a sum, but that does involve a recursive operator and so does not correctly answer the question.

Some students lost a mark for the step case of the induction because they did not put down any justifications, or because their justifications did not fit the equality they were put against, and some lost a second mark by writing a proof that looked as if they were assuming what they wanted

to show (although in many cases a bit more care would have rescued the given argument).

- b) The majority of students could give the correct definition of the desired function, which should return the empty list in the base case, and $1 : (s : \text{ins}_1 l)$ in the step case. Many students left out the brackets in the step case, but that was not penalized. Some students wrote incorrect syntax for lists ($[1, s, \text{ins}_1 l]$ is not a well-formed list). Those who wrote $[1, s] ++ \text{ins}_1 l$ did get full marks in this part but they invariably lost marks in the proof because they could not justify correctly the result of applying the sum operation to that expression.

For the code the first return clause should return **null** (or the argument l) in the first and

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new List (1, new List (l.value, ins1(l.next)));
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in the second case, and this latter was often answered wrongly. By and large the students who tried the induction proof could carry it out correctly, although there were quite a few incomplete base cases (evaluating $\text{sum ins}_1 []$ to 0, but stopping there instead of looking at the other side of the equality), and a few whose justifications for the various steps were missing or wrong.

- c) This was meant to be tricky for the last few marks. There are two possible proofs. There is one by induction (which needs to start with 0 rather than 1 to cover all natural numbers), whose step case relies on correctly calculating the coefficients in

$$(n + 1)^5 = n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1,$$

and realizing that all the inner terms are equal to 0 when calculating $\text{mod}5$, and then the induction hypothesis gives the desired result.

The second variant is to work out that there are only 5 equivalence classes when calculating $\text{mod}5$, and it is possible to calculate the fifth power for all those. A lot of students who seemed to try something along these lines did not really explain what they were doing and it was rare to be awarded full marks for this approach.

Question 2. Overall this was quite well answered apart from part b) where many students received no marks at all, and the very last part of d), which was meant to be technically difficult.

- a) A lot of students seemed to miss the fact that binary strings consist of symbols 0 and 1 exclusively, and that should make some of the arguments much simpler than the ones that I encountered. Some students spoke of 'lists' and used that syntax.

The first relation is an equivalence relation, and three marks were awarded for stating this correctly and making the argument (which is quite a simple one, but some students didn't give an argument, or theirs was incoherent). Most students wrote that the equivalence class of 1, which I asked for, consisted of all the strings who, interpreted as a binary number, were equal to 1, but that's just the definition. I was looking for a description of these strings, and to get the mark for this part I needed to see something along the lines of trying to list the elements as

1, 01, 001, 0001, ...

or, more elegantly, describe the set as

$$[1] = \{0^n 1 \mid n \in \mathbb{N}\}.$$

The second relation isn't an equivalence relation since it is neither reflexive nor transitive. I was quite generous in awarding marks for the argument here—very few students just picked some strings to demonstrate the problem, but instead they argued using variables, clearly having forgotten that the underlying set of symbols has only two elements.

- b) The given relation is not anti-symmetric, and a lot of students missed this completely. They then drew a diagram which did not represent the given relation. I had specifically given some strings for which anti-symmetry fails (011 and 101) as a bit of a hint, but a lot of students got this wrong nonetheless, and they received none of the three available marks for this part.
- c) A lot of students missed the fact that the tasks run from A to F (and the last task is not related to any other), and so their answer missed this element everywhere. Some students got the relation the wrong way round. I tried not to penalize students repeatedly for the same mistake here. A lot of students drew a diagram with arrows, whereas a Hasse diagram does not have any arrows. When this is done correctly it is quite easy to see that there are three minimal elements, and so there can't be a least element. It is not true that tasks which rely on others can't be designed and implemented at the same time (some programs would never get finished if that were the case!), so getting the final mark here required something different. Any statement that is true and relates to the given order got that mark.
- d) Some students forgot to draw the trees, but everybody who did so did it correctly. The trees that are related by the given relation are the first and second given tree, and the third and the fifth, reading from left to right and then down.

A characterization of the given relation in a non-recursive way is tricky, and nobody got full marks for this part. It requires defining a bijection between the nodes of the two trees in such a way that it preserves labels and restricts to bijections for each level of the tree. (There are other ways of putting this correctly.) Marks were awarded depending on how close students came in characterizing the relation. (Note that stating that the second tree has to result from the first by swapping out sub-trees is correct, but is effectively still a recursive procedure at best.) Many students seemed to assume that labels were unique (and this is true for the trees that were given as examples), but everybody should have learned during the examples classes that labels can occur multiple times in a tree.

Question 3. Compared to the semester 1 exam and also last years exam the quality of the answers was generally much better and fewer of the answers to this question seemed rushed.

- a) Average mark: 37%. 31 students achieved full marks.

Most students struggled with this question. Common problems were:

- struggling to understand the question (the question needed to be read carefully and drawing a picture will have helped)
- The vector from A to B is $\underline{b} - \underline{a}$, and not $\underline{a} - \underline{b}$.

The intention was to derive a formula for i) and then substitute the coordinates of points A and B for the vectors \underline{a} and \underline{b} to get the answer for ii). Some students computed the correct answer for ii) independently by other means which also earned full marks.

- b) Average mark: 85%. 149 students achieved full marks.

This questions posed few problems. It required being able to multiply matrices and knowing that \underline{I}_2 is the 2×2 identity matrix.

Note that for ii), since $2\underline{A}2\underline{B} = 4\underline{AB}$, the answer form i) could be used in ii), making the calculation easy if one knew what \underline{I}_2 is. Correct answers with long calculations received full marks.

- c) Average mark: 64%. 109 students achieved full marks.

I was looking for a short description in words of the geometric interpretation of the transformation. A few students correctly said that a typical vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is mapped to $\begin{pmatrix} x \\ -y \end{pmatrix}$ but then said this means rotation by 90 degrees (or something else), which is not correct. This earned 1/2 marks. Some students drew a picture, which suggested a rotation operation so could not earn full marks.

- d) Average mark: 46%. 1 student achieved full marks.

Part i) posed few problems and most students got this one right and gave very nice arguments!

Part ii) required to be carefully read. The question required giving an example of two matrices \underline{A} and \underline{B} be given such that $\underline{AB} = \underline{BA}$ does hold. A good range of different correct answers were given. Some students misunderstood this as a question about non-commutativity of matrix multiplication, which it wasn't.

Part iii) was basically a book work question and just required stating one of the main theorems we established in class.

Part iv) was the hardest question. It required giving a short argument or proof which about half a dozen students managed to do. One possibility was to note what happens when reducing the matrices $\underline{A}|\underline{0}$ and $\underline{A}|\underline{b}$ to RREF form and noting that the RREFs are $\underline{I}|\underline{0}$ and $\underline{I}|\underline{b}'$, respectively. Another possibility was to formally show: if $\underline{Ax} = \underline{b}$ has two solutions \underline{v} and \underline{w} then $\underline{v} = \underline{w}$.

In each case, 1 mark was awarded for a correct True/False answer and a correct argument or counter-example. This means correct guesses with wrong or absent explanations would have received 1 mark.

- e) Average mark: 77%. 100 students achieved full marks

Almost everyone got Part i) right. Some students wrote the vector equation representation of the system, but this is not what I was looking for.

Part ii) was generally well-answered. Marks were typically lost due to:

- small mistakes in the calculations
- not using the Gaussian elimination algorithm and manipulating equations directly
- drawing the wrong conclusion from the reduced augmented matrix; that there are infinitely many solutions is the wrong conclusion.