Special instructions: You may work either individually or in a group of exactly two persons. Write your solutions out on paper and deliver them to SSO by 15:00 on Friday, 30th November, 2012. If you are working in a group, one script should be submitted for the group, with both names at the top; both members of the group will then receive the same mark. Clearly write your name(s), student ID number(s) and the words "Comp36111 Sec. B Coursework" on the front (cover) sheet and staple all sheets together. Submitted solutions must be entirely the effort of the persons whose names appear at the top of the submission.

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms I: Coursework for Sec. B

Time: This should take you a few hours

Please answer all questions.

Marks will be awarded for clarity and succinctness as well as correctness. If you cannot solve Question 4, you may describe a failed attempt.

The use of electronic calculators is not recommended.

Section B

1. Explain what is meant by a *logarithmic space, many-one reduction*. (2 marks)

2. Define the problem 3-COLOURABILITY for (undirected) graphs. What do you know about the computational complexity of this problem? (4 marks)

3. Let $\mathcal{N}^\circ$ be the set of sentences in first-order logic with counting, having the forms

$$\exists_{\geq C} x(p(x) \wedge q(x)) \qquad \exists_{\leq C} x(p(x) \wedge q(x)),$$

   where p and q are unary predicates, and C is a natural number. We interpret these statements in the expected ways:

$\exists_{\geq 13} x(\text{artist}(x) \wedge \text{beekeeper}(x))$	$\exists_{\leq 0} x(\text{beekeeper}(x) \wedge \text{carpenter}(x))$
At least 13 artists are beekeepers	At most 0 beekeepers are carpenters

   A collection Φ of such statements is said to be *satisfiable* if it is possible to make all sentences in Φ true together.

   Write a collection Φ of four such formulas, such that Φ is not satisfiable, but any proper subset of Φ is. (Hint: you don't need big numbers.)

   (4 marks)

4. Suppose now we are given a finite graph $G = (V, E)$, where $V = \{1, \ldots, n\}$. For all i ($1 \leq i \leq n$) and k ($0 \leq k < 3$), let p_i^k be a unary predicate, and let p be a further unary predicate. Think of $p_i^k(x)$ as saying: "x is a colouring of G in which node i has colour k". (I will leave you to interpret p yourselves.) Let Φ_G be the set of $\mathcal{N}^\circ$-formulas consisting of

$$\exists_{\leq 3} x(p(x) \wedge p(x))$$
$$\{\exists_{\leq 0} x(p_i^j(x) \wedge p_i^k(x)) \mid 1 \leq i \leq n, 0 \leq j < k < 3\}$$
$$\{\exists_{\geq 1} x(p_i^k(x) \wedge p(x)) \mid 1 \leq i \leq n, 0 \leq k < 3\}$$
$$\{\exists_{\leq 0} x(p_i^k(x) \wedge p_j^k(x)) \mid (i, j) \text{ is an edge of } G, 0 \leq k < 3\}$$

   Show that Φ_G is satisfiable if and only if G is 3-colourable.

   (8 marks)

5. What does this result tell us about the computational complexity of determining satisfiability of statements in $\mathcal{N}^\circ$?

   (2 marks)

END OF EXAMINATION