COMP20010 - Math and Complexity Practice Questions

April 23, 2010

1 Big-O

1.1 Practice

What is big-O complexity of the following?

1. $100N + N^2$
2. $3N^5 + 2N + 13N^2 + 9$
3. $\sqrt{N} + 3N + 100$
4. $10^6N + N^2$
5. $\frac{N}{1+N}$
6. $\sin N$

1.2 Loops within algorithms

What is the big-O complexity of the following methods?

Algorithm1 : Compute mean and variance of an array size N

```
mean ← 0
var ← 0
{Compute mean}
for i = 1 to N do
  mean ← mean + array[i]
end for
{Compute variance}
for i = 1 to N do
  var ← var + (array[i] - mean)*(array[i] - mean)
end for
print mean, var
```

Algorithm 2:

```
for i = 1 to N do
  for j = 1 to 10 N do
    print i + j
  end for
end for
```

Algorithm 3:

```
for i = 1 to N do
  if factorial(i) > 100 then
    return i
    break
  end if
end for
```

Here `factorial()` is a method which computes the factorial of an number, i.e. `factorial(n)` $= 1 \times 2 \times \cdots \times n$.

Algorithm 4:

```
for i = 1 to N do
  for j = 1 to 10N do
    for k = 1 to N do
      print  k * (i + j)
    end for
  end for
end for
```

Algorithm 5:

```
for i = 1 to N do
  for j = 1 to i do
    print  i + j
  end for
end for
```

Algorithm 6:

```
for i = 1 to N do
  temp ← list[i]
  list[i] ← 0
  sort(list)
  list[i] ← temp
end for
```

where `sort` is an $O(N \ln N)$ sorting algorithm.

1.3 Exponential Growth

1.4 Logarithms

Find the following logarithms (without using a calculator).

1. $\log_2 128$
2. $\log_3 9$
3. $\log_{10}(10^5)$
4. $\log_{100} 100$
5. $\log_{10} 50$ lies between what two integers? Is it halfway between?

Let `database` be a database of names and phone numbers stored in an array, where the array is alphabetically by names. In other words, if $i < j$, then `database[i].name`<=`database[j].name`, where here less than means in alphabetical order.

The following is an algorithm to find the phone number of a particular name. What is the big-O complexity of this algorithm? (`sname` is the string containing the name being searched for.)

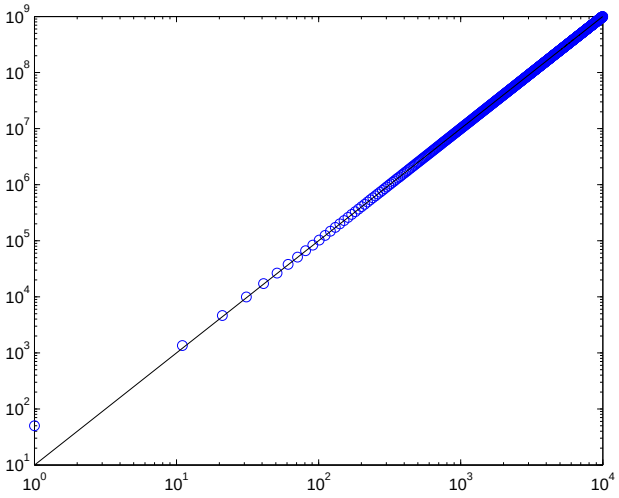
Algorithm BinarySearch

```
min ← 1
max ← N
repeat
  mid ← (min + max) div 2
  if sname > database[mid].name then
    min ← mid +1
  else
    max ← mid −1
  end if
until (database[mid].name = sname) or(min > max);
if (min > max) then
  print ''name not found''
else
  print database[mid].number
end if
```

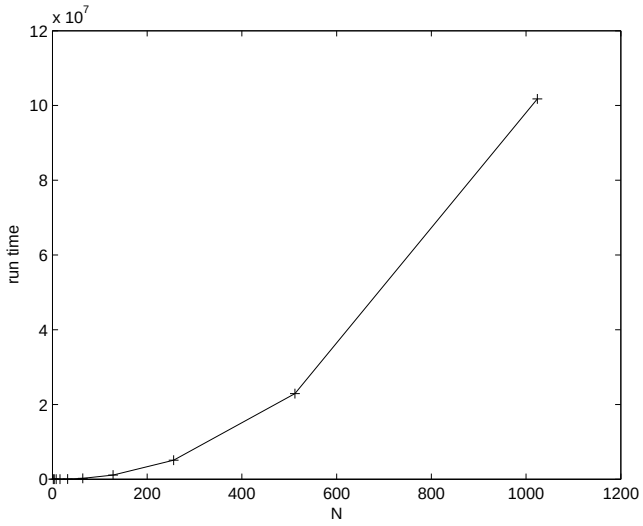
Hint: How big is the search interval at the start of each iteration of the `repeat` loop? How many times does the `repeat` loop run.

1.5 Log-log plots

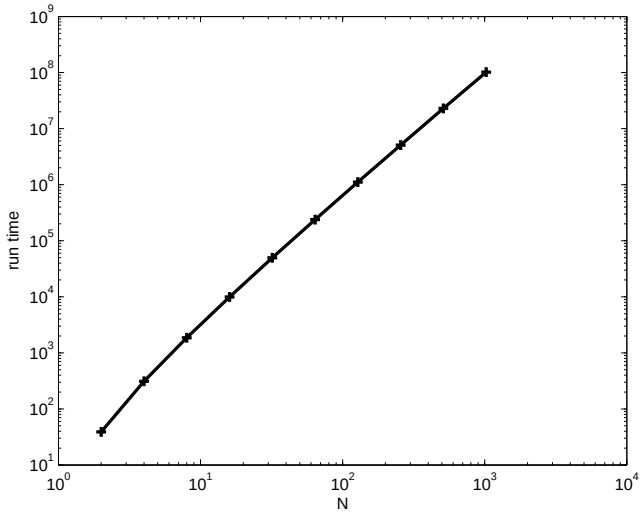
1. The plot below shows some data plotted on a log-log plot, and a best-fit line. Is it a power law? What is the formula?



2. The plot below shows the results of some experiments on run times. This could follow a powerlaw.



The plot below shows the same data plotted on a log-log plot. Does it exactly follow a power law. Give an approximate formula for the power law for large N.



3. Here is the data from the January Mock exam. Give the formula of the best fit powerlaw.