Exercise 1. Simulations
Consider the two automata below.

- Is there a simulation from X to Y? If so, provide it. If not, give an argument why there cannot be.
- Is there a simulation from Y to X? If so, provide it. If not, give an argument why there cannot be.
- Do the automata accept the same language?

X

\[ A \xrightarrow{a} B \xleftarrow{b} A \]

Y

\[ 0 \xrightarrow{a} 1 \xleftarrow{b} 0 \]

Exercise 2. Regular Languages
Which of the following languages are regular? Provide a justification in each case.

(a) The language of all words over \( \{0, 1\} \) that have odd length.

(b) The language of all words over \( \{0, 1\} \) that have odd length and a 1 in the middle.

(c) The language of all strings over the alphabet \( \{0, 1\} \) which, when viewed as a binary number, are prime numbers.

(d) The language of all non-empty strings over the alphabet \( \{0, 1, \ldots, 9\} \) such that the last symbol is one that occurred earlier in the string.

(e) The language of all strings over \( \{0, 1\} \) where the number of 0 is different from the number of 1s.

(f) The language of all palindromes over \( \{0, 1\} \), that is, all strings that read forwards as backwards or, to put it differently, the strings which are equal to their reversal.

(g) The language of all strings over \( \{0, 1\} \) such that there are two 0s in the string which are separated by a number of characters that is a non-zero multiple of 4.

(h) The language of all odd-length palindromes over \( \{a, b, c\} \).
(i) The language of all words over the alphabet \{a, b\} that have the same number of occurrences of the substring \(ab\) as that of the substring \(ba\). The word \(bab\) does belong to that language, since it has one occurrence of the substring \(ab\) ([ab]a) and one of the substring \(ba\) (a[ba]). *Does your answer change when the alphabet is changed to \{a, b, c\}?

(j) The set of all words of the form \(s1^n\), where \(s\) is a word over \{a, b, c\}, and \(n\) is the number of letters in \(s\).

(k) The set of all words over the alphabet \(a, b, 0, 1\) which consist of blocks of \(as\) and \(bs\) followed by a number 0 or 1, where the number is 0 if the block has even length, and is 1 if the block has odd length. For example, the words \(aba1bbaa0\) and \(0abab0b1\) are in the language, but \(ab1\) and \(ab0baa0\) are not.