Special instructions: You may work either individually or in a group of exactly two persons. Write your solutions out on paper, photocopy them and hand in both the original and the copy to SSO by 12:00 on Thursday, 19th October, 2017. If you are working in a group, one script should be submitted for the group, with both names at the top: both members of the group will then receive the same mark. Clearly write your name(s), student ID number(s) and the words “Comp36111 Sec. A Coursework” on the front (cover) sheet and staple all sheets together. Submitted solutions must be entirely the effort of the persons whose names appear at the top of the submission.

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Advanced Algorithms I: Coursework for Sec. A

Time: This should take you a few hours

Please answer all questions.
Marks will be awarded for clarity and succinctness as well as correctness.

The use of electronic calculators is not recommended.
The School of Computer Science at Manchester University has decided to subject all Third Year Undergraduates, \( S_1, \ldots, S_n \), to additional oral examinations. The examinations will all take place on a certain day between the hours 9:00 and 17:00 and will each last 37 minutes. In order to provide an element of surprise, however, it was decided that the times of these examinations would be randomly (and independently) chosen: Student \( S_i \)’s examination begins at time \( s_i \) (a time in whole minutes between between 9:00 and 16:23 inclusive). These times have now been chosen and, by University Statute, cannot be changed. Examinations of different students may overlap or even coincide completely. The School has the task of finding rooms for these examinations. Each examination is conducted in one of a large number of rooms \( R_1, R_2, \ldots \). Of course, no two students can be examined simultaneously in the same room. There are plenty of rooms, as the University can always rent more space. Nevertheless, mindful of the expense, the School wishes to minimize the number of rooms used.

1. Give a polynomial time algorithm which takes the start times \( s_1, \ldots, s_n \) as input and allocates examination rooms to students so as to (avoid any clashes and) use the minimum number of rooms. (6 marks)

2. What is the asymptotic complexity of your algorithm, as a function of \( n \) (the number of students)? (2 marks)

3. Give a careful proof that your algorithm never uses more rooms than is absolutely necessary. (8 marks)

4. It occurred to the School Senior Leadership Team only at the last minute that they also needed to schedule Examiners to conduct these oral examinations. One and only one examiner is required for each exam. As with rooms, extra Teaching Assistants can always be hired, so there is no danger of running out; but again, the School would like to minimize costs. Examiners can be teleported instantaneously between rooms if necessary; but no examiner can be in two different rooms at once for any non-zero amount of time. Can we get away with fewer examiners than there are rooms? Justify your answer carefully. (4 marks)