UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science
04/11/14
Time: 12.00

This is a CLOSED book examination

The use of electronic calculators is not permitted.
1. a) Consider the following function:

\[ f: \mathbb{Z} \rightarrow \mathbb{N} \]
\[ x \mapsto \begin{cases} 
3x & \text{if } x \geq 0 \\
-3x + 1 & \text{otherwise}
\end{cases} \]

Is this function injective? Is it surjective? Justify your answers. (5 marks)

b) Consider the following binary operation on \( \mathbb{C} \): We set

\[ z \odot z' = z \cdot z' + z + z'. \]

Is this operation associative? Is it commutative? Justify your answers. (5 marks)

2. a) Show

\[ (P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P) \]

in the Boolean semantics by using truth tables. (3 marks)

b) Give a brief explanation of one of the following. (2 marks)

i. subformula
ii. Boolean valuation
iii. substitution for a propositional variable

c) Consider the following propositional formula. (5 marks)

\[ (\neg P \lor Q) \rightarrow (P \rightarrow (P \rightarrow Q)) \]

i) Give a conjunctive normal form for the formula. Simplify as far as possible.
ii) Give a disjunctive normal form for the formula.