MANCHESTER 1824

The University of Manchester

Week 4

COMP62342

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Today

- Some clarifications from last week's coursework
- More on reasoning:
  - extension of the tableau algorithm & discussion of blocking
  - traversal or "how to compute the inferred class hierarchy"
  - OWL profiles
- The OWL API: a Java API and reference implementation for
  - creating,
  - manipulating and
  - serialising OWL Ontologies and
  - interacting with OWL reasoners
- **Lab**:
  - OWL API for coursework
  - Ontology Development

Some clarifications from last week's coursework

Ontologies, inference, entailments, models

- *OWL* is based on a *Description Logic*
  - we can use DL syntax
  - e.g., C ⊑ D for C SubClassOf D

- An OWL ontology O is a **document:**
  - therefor, it cannot **do** anything - as it isn't a piece of software!
  - in particular, an ontology cannot **infer** anything
    (a reasoner may infer something!)

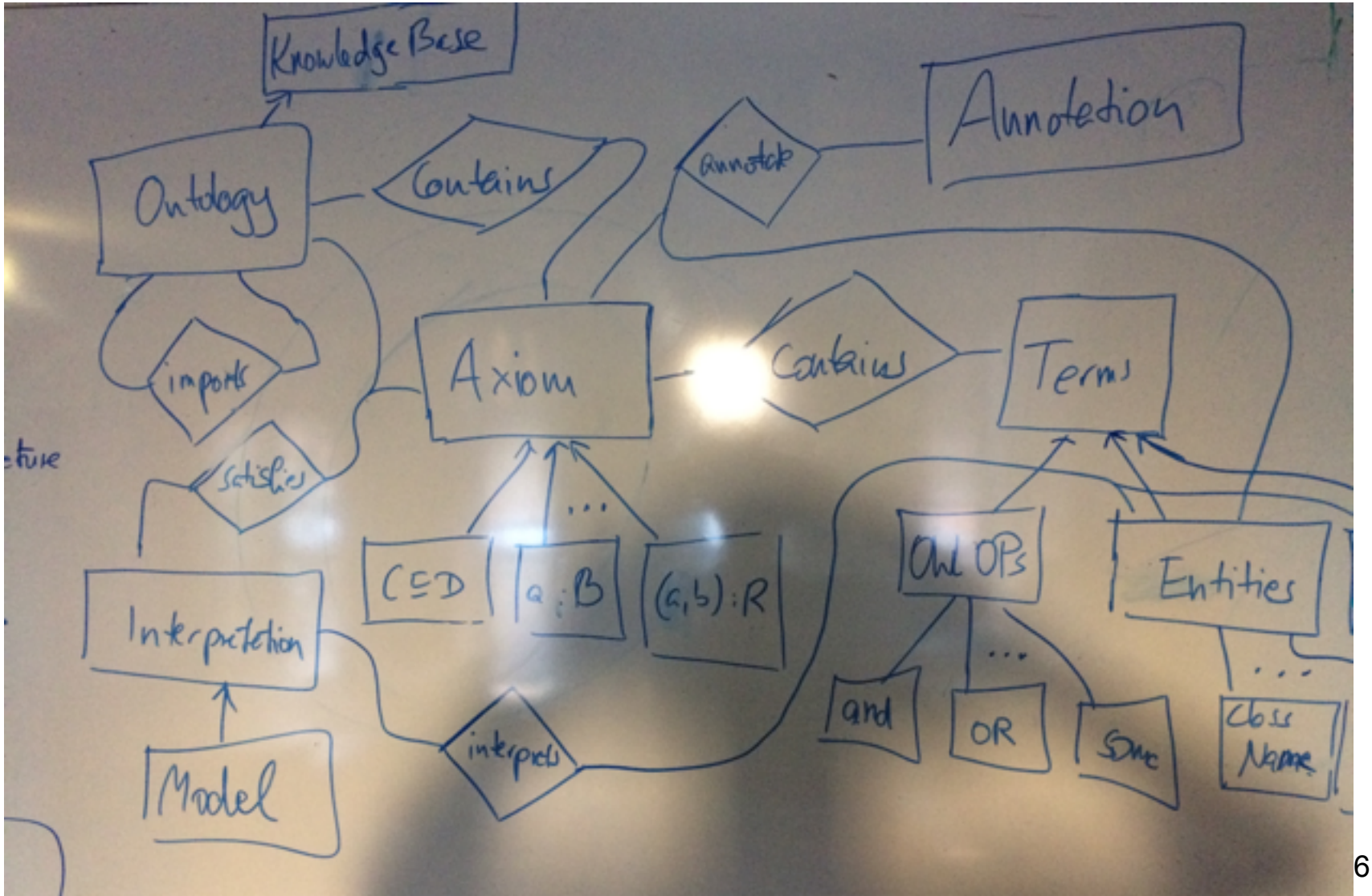
- An OWL ontology O is a **web document:**
  - with 'import' statements, annotations, …
  - corresponds to a **set of logical OWL axioms**
  - the OWL API (today) helps you to
    - parse an ontology
    - access its axioms
  - a **reasoner** is only interested in this set of axioms
    - **not** in annotation axioms
    - see https://www.w3.org/TR/owl2-primer/#Document_Information_and_Annotations
    - https://www.w3.org/TR/2012/REC-owl2-syntax-20121211/#Annotations

Ontologies, inference, entailments, models (2)

- We have defined what it means for O to **entail** an axiom C SubClassOf D
  - written O $\models$ C SubClassOf D
    or O $\models$ C $\sqsubseteq$ D
  - based on the notion of a **model** I of O
    - i.e., an **interpretation** I that satisfies all axioms in O
  - don't confuse 'model' with 'ontology'
    - one ontology can have **many** models
    - the more axioms in O the fewer models O has

- A **DL reasoner** can be used to
  - check entailments of an OWL ontology O and
  - compute the **inferred class hierarchy** of O
    - this is also known as **classifying** O
  - e.g., by using a **tableau algorithm**

Learn terms & meaning & relations!



More on Reasoning

Recall Week 2: OWL 2 Semantics: Entailments

Let O be an ontology, α an axiom, and A, B classes, b an individual name:

- O is **consistent** if there exists some model I of O
  - i.e., there is an interpretation that satisfies all axioms in O
  - i.e., O isn't self contradictory
- O **entails** α (written $O \models \alpha$) if α is satisfied in all models of O
  - i.e., α is a consequence of the axioms in O
- A is **satisfiable** w.r.t. O if $O \not\models$ A SubClassOf Nothing
  - i.e., there is a model I of O with $A^I \neq \{\}$
- b is an **instance of** A w.r.t. O (written $O \models b{:}A$) if $b^I \subseteq A^I$ in every model I of O

Theorem:

1. O is consistent iff $O \not\models$ Thing SubClassOf Nothing
2. A is satisfiable w.r.t. O iff $O \cup \{n{:}A\}$ is consistent (where n doesn't occur in O)
3. b is an instance of A in O iff $O \cup \{b{:}not(A)\}$ is not consistent
4. O entails A SubClassOf B iff $O \cup \{n{:}A \text{ and } not(B)\}$ is inconsistent

Recall Week 2: OWL 2 Semantics: Entailments etc.

Let O be an ontology, α an axiom, and A, B classes, b an individual name:

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- b is an **instance of** A w.r.t. O if $b^I \subseteq A^I$ in every model I of O

- O is **coherent** if every class name that occurs in O is satisfiable w.r.t O
- **Classifying O** is a reasoning service consisting of
  1. testing whether O is consistent; if yes, then
  2. checking, for each pair A,B of class names in O plus Thing, Nothing
     $O \models$ A SubClassOf B
  3. checking, for each individual name b and class name A in O, whether $O \models$ b:A

     …and returning the result in a suitable form: O's **inferred class hierarchy**

Week 3: how to test satisfiability …

Last week, you saw a **tableau algorithm** that

- takes a class expression C and decides **satisfiability** of C
- i.e., it
  - answers 'yes' if C is satisfiable
    'no' if C is not satisfiable
  - sound, complete, and terminating

- we saw this for the $\mathcal{ALC}$
  - $\mathcal{ALC}$ is a Description Logic that forms logical basis of OWL
  - only and, or, not, some, only
- works by trying to generate an interpretation with an instance of C
  - by breaking down class expressions (in NNF!)
  - generating new P-successors for some-values from restrictions
    (∃P.C restrictions in DL)

- we can handle an ontology that is a set of **acyclic SubClassOf axioms**
  - via **unfolding** (check Week 3 slides!)

Week 3: tableau rules

$x \bullet \{C_1 \sqcap C_2, \ldots\}$	$\rightarrow_{\sqcap}$	$x \bullet \{C_1 \sqcap C_2, C_1, C_2, \ldots\}$
$x \bullet \{C_1 \sqcup C_2, \ldots\}$	$\rightarrow_{\sqcup}$	$x \bullet \{C_1 \sqcup C_2, C, \ldots\}$ For $C \in \{C_1, C_2\}$
$x \bullet \{\exists R.C, \ldots\}$	$\rightarrow_{\exists}$	$x \bullet \{\exists R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{C\}$
$x \bullet \{\forall R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{\ldots\}$	$\rightarrow_{\forall}$	$x \bullet \{\forall R.C, \ldots\}$ $\xrightarrow{R}$ $y \bullet \{C, \ldots\}$

Mini-exercise

- Apply the tableau algorithm to test whether A is satisfiable w.r.t.

$\{A \sqsubseteq B \sqcap \exists P.C,$
$B \sqsubseteq C \sqcap \forall P.(\neg C \sqcup D)\}$

{A SubClassOf B and (P some C),
A SubClassOf C and (P only (not C or D)}

This week: GCIs and tableau algorithm

- When writing an OWL ontology in Protégé,
  - axioms are of the form A SubClassOf B with A a class **name**
  - (or A EquivalentTo B with A a class **name)**

  - last week's tableau handles these via **unfolding:**
    - works only for **acyclic** ontologies
    - e.g., not for A SubClass (P some A)

- **but** OWL allows for **general class inclusions (GCIs),**
  - axioms of the form A SubClassOf B with A a class **expression**
  - e.g., (eats some Thing) SubClassOf Animal
  - e.g., (like some Dance) SubClassOf (like some Music)
  - this requires another rule:

$$x \bullet \{\ldots\} \quad \rightarrow_{GCI} \quad x \bullet \{\neg C \sqcup D, \ldots\}$$

for each $C \sqsubseteq D \in O$

GCIs and tableau algorithm

$x \bullet \{\ldots\} \quad \rightarrow_{GCI} \quad x \bullet \{\neg C \sqcup D, \ldots\}$

for each $C \sqsubseteq D \in O$

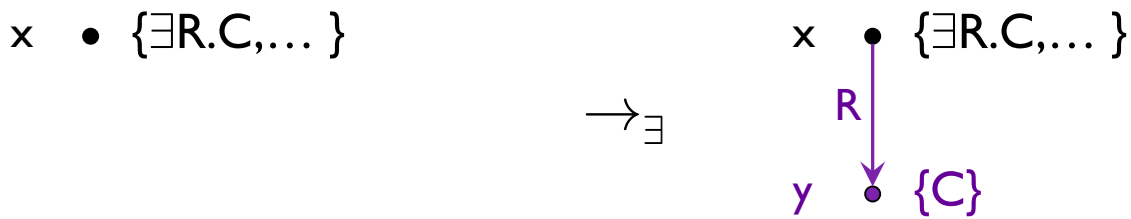
- E.g., test whether

  A is satisfiable w.r.t.
  {A SubClassOf (P some A)}
  or $\{A \sqsubseteq \exists P.A\}$

- This rule easily causes **non-termination**
  - if we forget to **block**

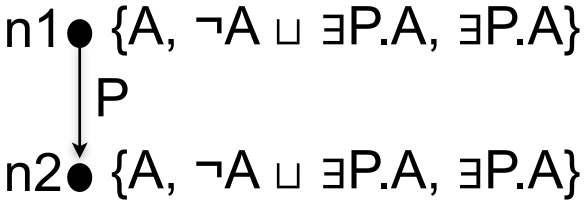
$\bullet \{A, \neg A \sqcup \exists P.A, \exists P.A\}$
$\xrightarrow{P}$
$\bullet \{A, \neg A \sqcup \exists P.A, \exists P.A\}$
$\xrightarrow{P}$
$\bullet \{A, \neg A \sqcup \exists P.A, \exists P.A\}$
$\xrightarrow{P}$
$\bullet \{A, \neg A \sqcup \exists P.A, \exists P.A\}$
$\xrightarrow{P}$
$\ldots$

Blocking



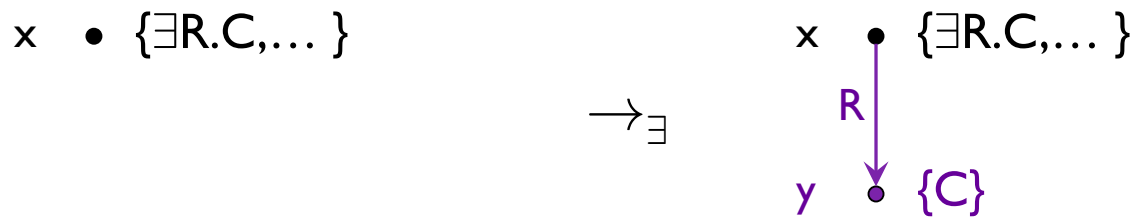
only if x's node label isn't contained in the node label of a predecessor of x

- Blocking ensures termination
  - even on cyclic ontologies
  - even with GCIs
- If x's node label is contained in the label of a predecessor y, we say "*x is blocked by y*"



- E.g., test whether A is satisfiable w.r.t. {A SubClassOf (P some A)}
  - here, n2 is blocked by n1

Blocking



only if x's node label isn't contained in the node label of a predecessor of x

- When blocking occurs, we can build a **cyclic** model from a complete & clash-free completion tree
  - hence soundness is preserved!

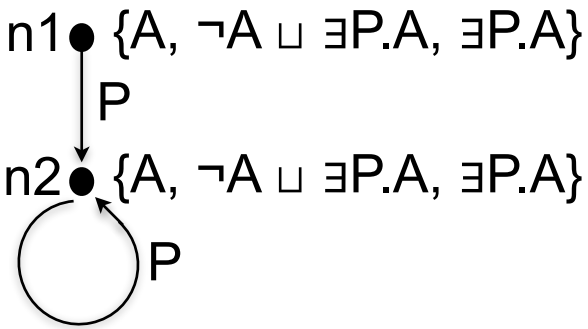


Tableau algorithm with blocking

Our ALC tableau algorithm with blocking is

- **sound**: if the algorithm stops and says “input ontology is consistent” then it is.
- **complete:** if the input ontology is consistent, then the algorithm stop and says so.
- **terminating:** regardless of the size/kind of input ontology, the algorithm stops and says
  - either “input ontology is consistent”
  - or “input ontology is **not** consistent”

- …i.e., a decision procedure for ALC ontologies
  - even in the presence of cyclic axioms!

Tableau algorithm & complexity

not too bad:
bounded by number of 'some' expressions in O

Our ALC tableau algorithm has a few sources of **complexity**

- the breadth/out-degree of the tree constructed
- the depth of/path length in tree constructed
- non-determinism due to $\rightarrow_{\sqcup}$ rule from
  - disjunctions in O, e.g., A SubClassOf B
  - SubClassOf axioms in O
  - EquivalentTo axioms in O

hopefully not too bad

2 disjunctions
per axiom in O
for each node in tree

- ok/li… r acyclic O
  …tial for
  …eneral O:
  …ct O
  …
  …each model has
  …a path of length 2^n

1 di…
i…

37,778,931,862,957,161,709,568

3 nodes with
25 SubClassOf axioms →
how many choices?

Tableau algorithm & complexity

- Without further details: deciding ALC satisfiability
  - only of class expressions is PSpace-Complete
  - of class expressions w.r.t. ontology is ExpTime-complete
  - …**much** higher than intractable/SAT

- Implementation of ALC or OWL tableau algorithm requires **optimisation**
  - there has been a lot of work in the last ~25 years on this
  - you see the fruits in Fact++, Pellet, Hermit, Elk, … reasoners available in Protégé
  - some of them from SAT optimisations, see COMP60332

- Next, I will discuss 1 optimisation: **enhanced traversal**

Naive Classification

- **Remember: Classifying O** is a reasoning service consisting of

1. testing whether O is consistent; if yes, then

   Test:
   is Thing satisfiable w.r.t. O?

2. checking, for each pair A,B of class names in O plus Thing, Nothing whether O ⊨ A SubClassOf B

   Test:
   is A ⊓¬B **un**satisfiable w.r.t. O?

3. checking, for each individual name b and class name A in O, whether O ⊨ b:A

   Test:
   is O ∪ {b:¬A} is **in**consistent?

…and returning the result in a suitable form: O's **inferred class hierarchy**

Naive Classification

- **Remember: Classifying O** is a reasoning service consisting of

1. testing whether O is consistent; if yes, then

Test: is Thing satisfiable w.r.t. O?	1 test

2. checking, for each pair A,B of class names in O plus Thing, Nothing whether O ⊨ A SubClassOf B

Test: is A ⊓¬B **un**satisfiable w.r.t. O?	n^2 tests for O with n class names

3. checking, for each individual name b and class name A in O, whether O ⊨ b:A

Test: is O ∪ {b:¬A} is **in**consistent?	nm tests for O with n class names, m individuals

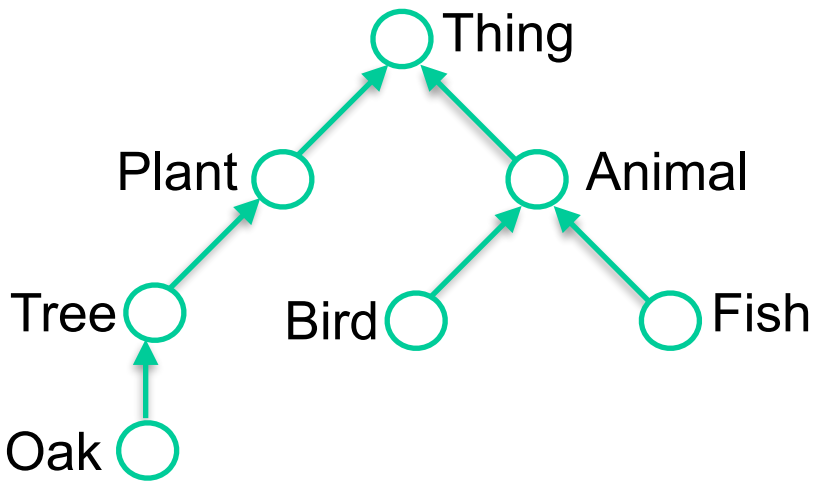
…and returning the result in a suitable form: O's **inferred class hierarchy**

Enhanced Traversal

- **Naive Classification of O** requires $1 + n^2 + nm$ expensive satisfiability/consistency tests
- …can we do better?

➡ Enhanced Traversal
  - idea: build inferred class hierarchy top-down and bottom-up, "trickling in" each class name in turn

- Assume you have, so far, constructed this hierarchy for O →



- Now you "trickle" Oak: check whether
  - $O \models Oak \sqsubseteq Plant$?
    yes - continue with Plant's child
  - $O \models Oak \sqsubseteq Animal$?
    no - ignore Animal's children!
  - $O \models Oak \sqsubseteq Tree$?
    yes - done!

  ➡ 2 entailment tests saved!

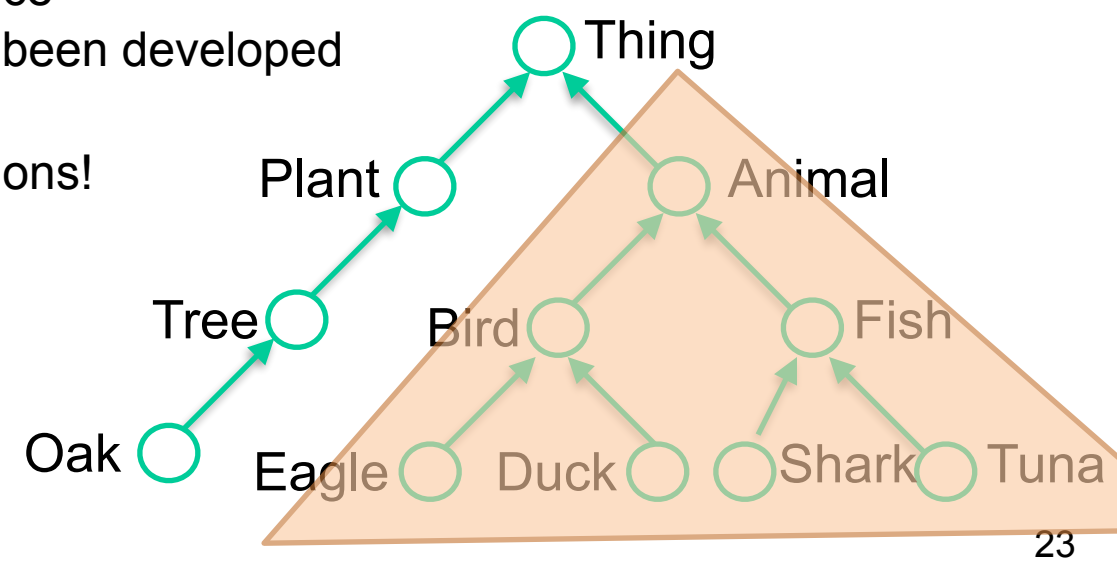
Enhanced Traversal

- **Naive Classification of O** requires $1 + n^2 + nm$ expensive satisfiability/consistency tests
- …can we do better?

➡ Enhanced Traversal
  - idea: build inferred class hierarchy top-down and bottom-up, "trickling in" each class name in turn

- Potentially avoids many of the n^2 satisfiability/consistency tests
  - very important in practice
  - different variants have been developed

- Just one of many optimisations!



OWL Profiles

- Despite all optimisations, classification of an ontology may still take too long if it is
  - big and/or
    - 300,000 axioms or more
  - rich
    - ALC plus inverse properties, atleast, atmost, sub-property chains,…

- For OWL 2 [*], **profiles** have been designed
  - syntactic fragments of OWL obtained by restricting constructors available

- Each profile is
  - **maximal,** i.e., we know that if we allow more constructors, then computational complexity of reasoning would increase
  - **motivated** by a use case

[*] the one we talk about here/you use in Protégé

OWL Profiles

In a nutshell, these are the profiles of OWL 2:

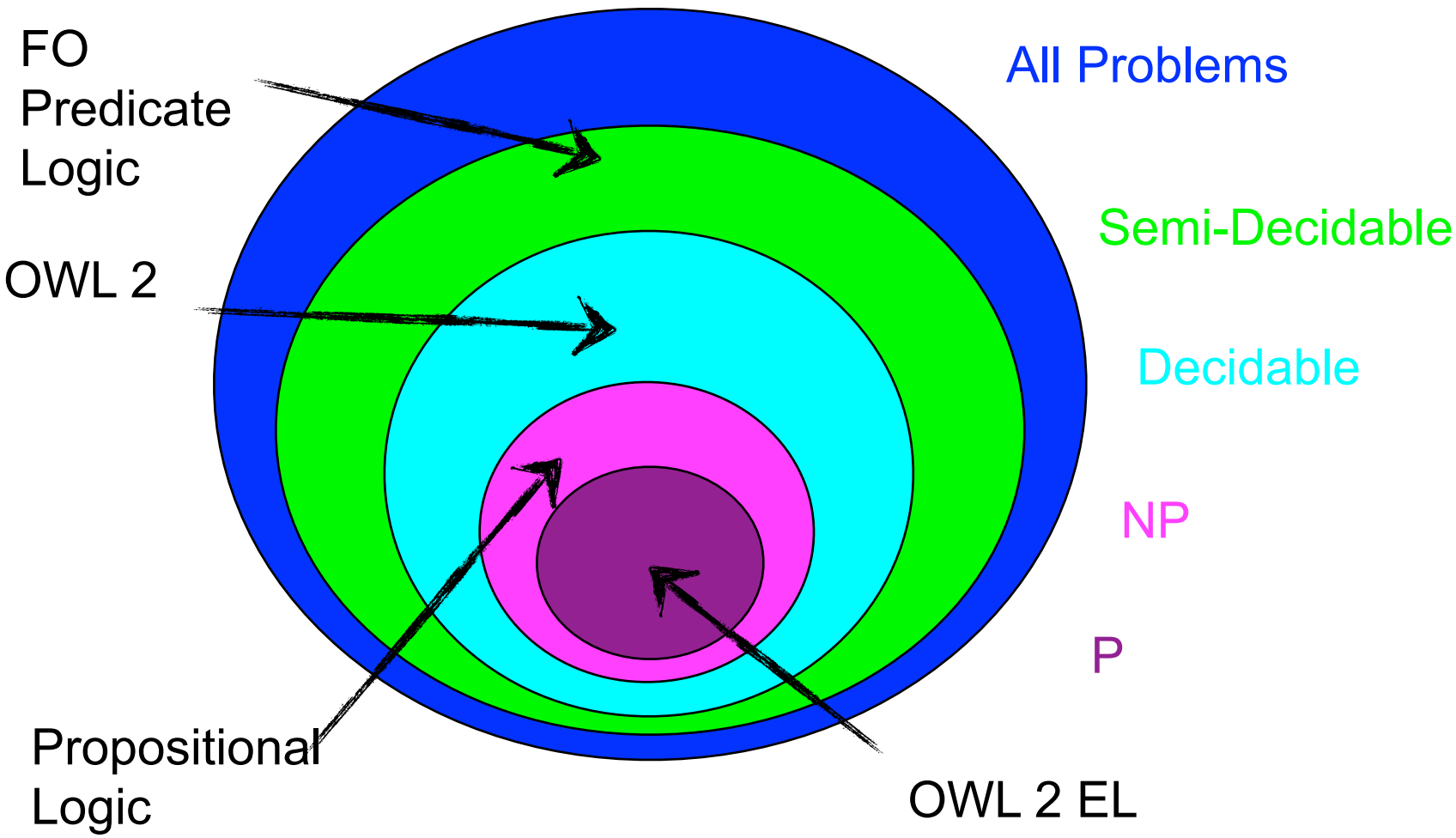
- OWL 2 EL:
  - only ‘and’, ‘some’, SubProperty, transitive, SubPropertyChain
  - it’s a *Horn* logic
    - no reasoning by case required,
    - no disjunction, not even hidden
  - designed for big class hierarchies

- OWL 2 QL:
  - only restricted ‘some’, restricted ‘and’, inverseOf, SubProperty
  - designed for querying data in a database through a class-level ontology

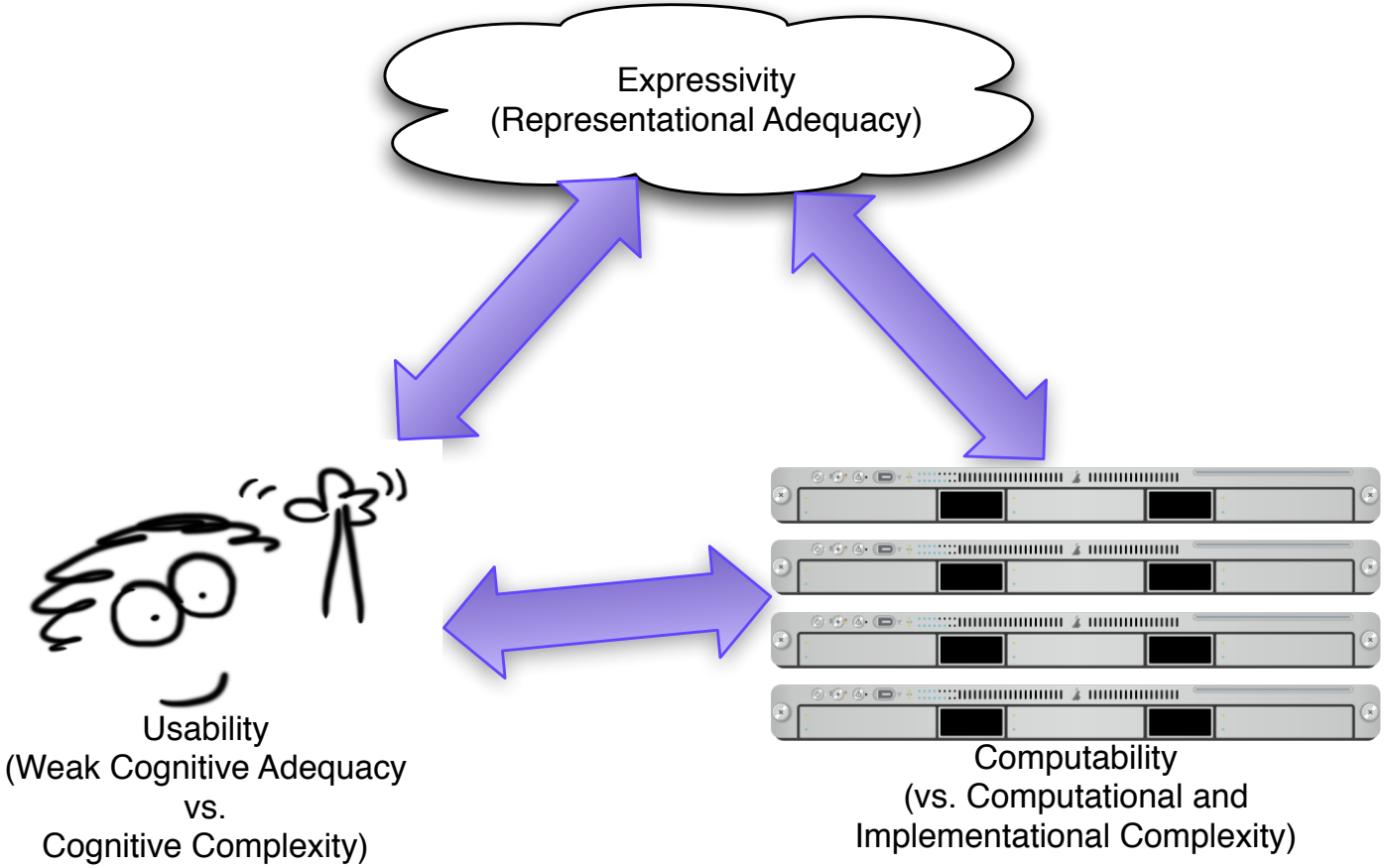
- OWL 2 RL:
  - no ‘some’ on RHS of SubClassOf, …
  - designed to be implemented via a classic rule engine

- For details, see OWL 2 specification!

Some Key Complexity Classes



The design triangle



Summary

OWL reasoning

- is unusual:
  - standard reasoning involves solving **many** reasoning problems/ satisfiability tests
- is decidable:
  - for standard reasoning problems, we have **decision procedures**
  - i.e., a calculus that is sound, complete, and terminating
- can be complex
  - but we know the complexity for many different DLs/OWL variants/profiles
  - and implementations require many good optimisations!

- goes beyond what we have discussed here
  - entailment explanation
  - query answering
  - module extraction
  - …

Today

- ✓ Some clarifications from last week's coursework
- ✓ More on reasoning:
  - ✓ extension of the tableau algorithm & discussion of blocking
  - ✓ traversal or "how to compute the inferred class hierarchy"
  - ✓ OWL profiles
- The OWL API: a Java API and reference implementation for
  - creating,
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